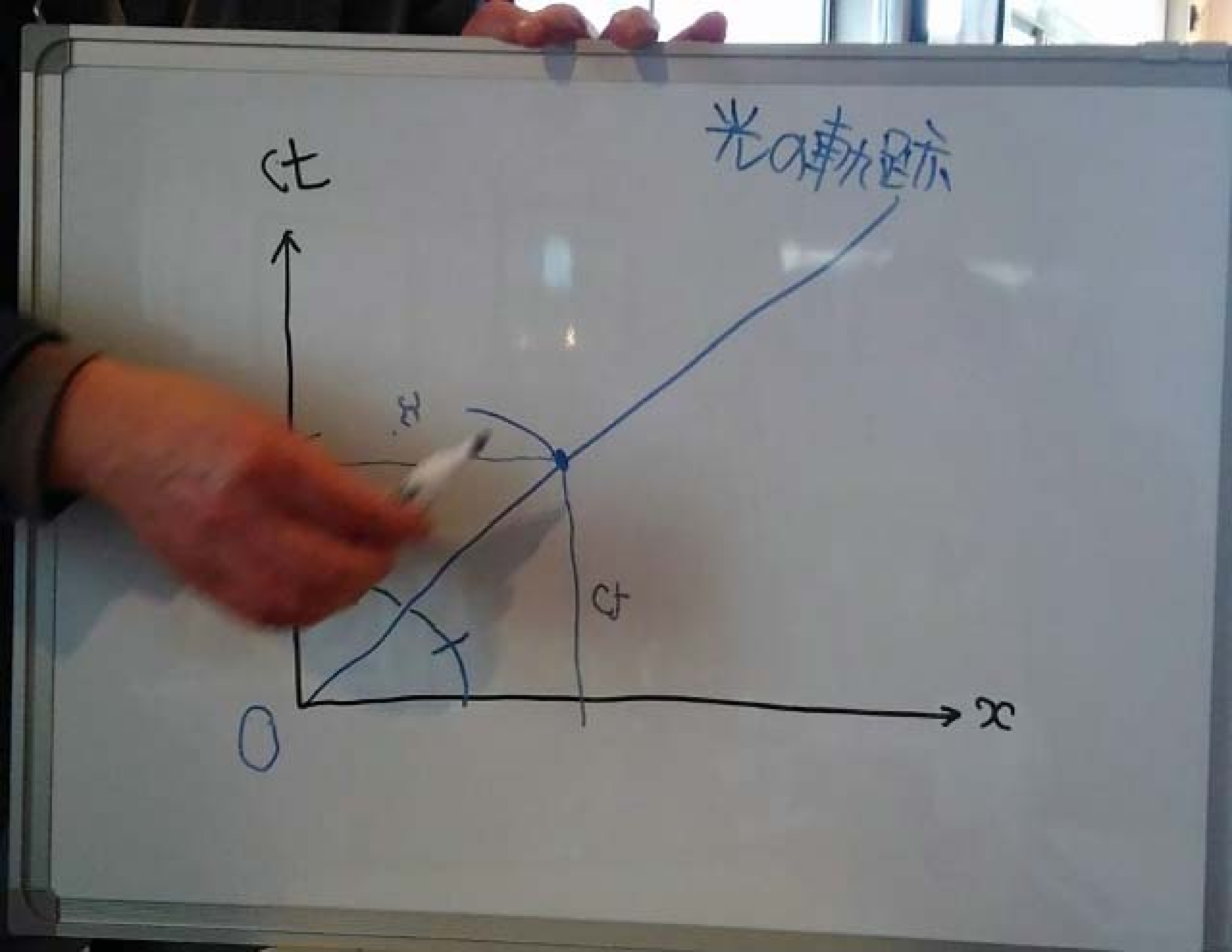
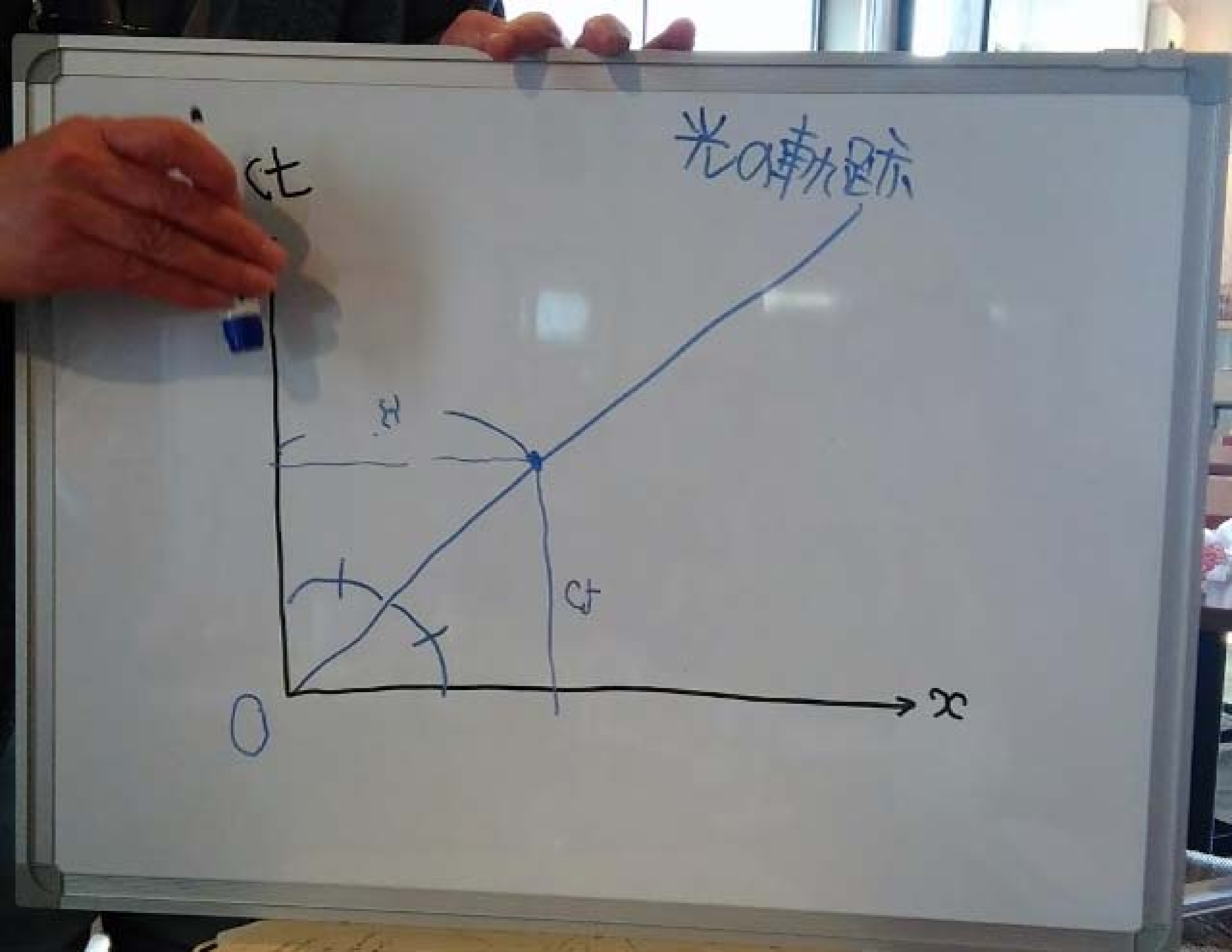
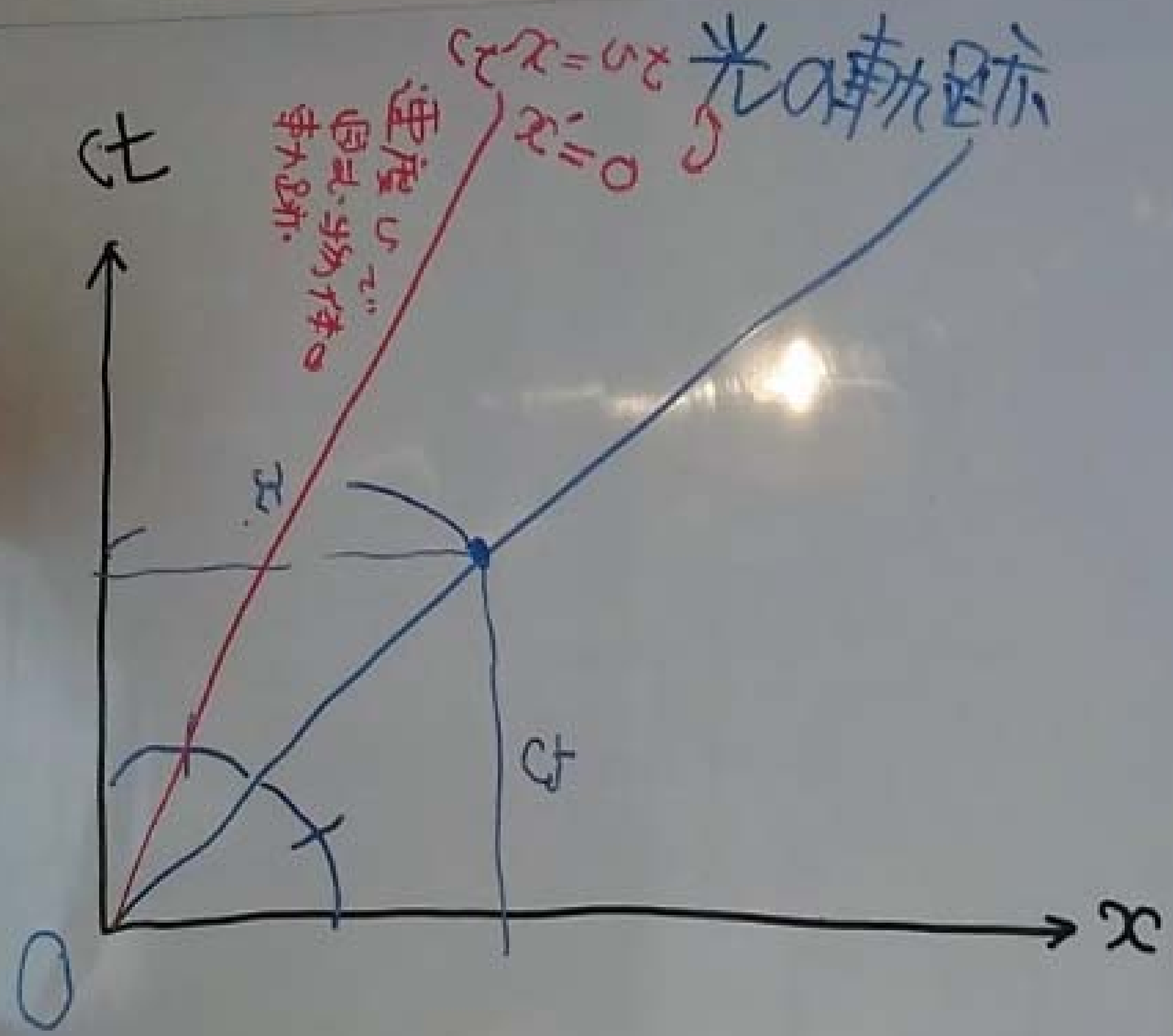
L1 特殊相対論

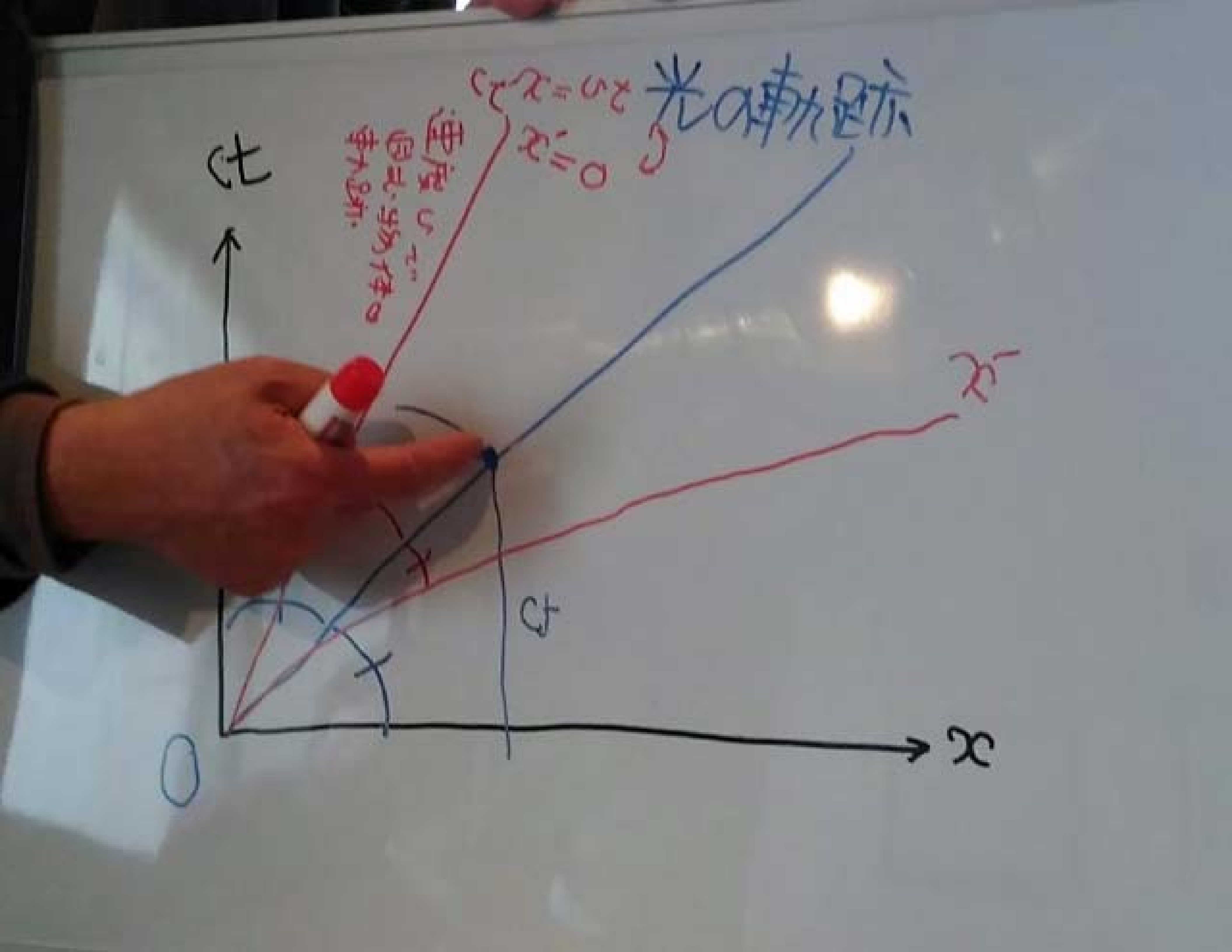
・光速不変の原理から
慣性系の間の関係
—— Lorentz変換を
みちびく

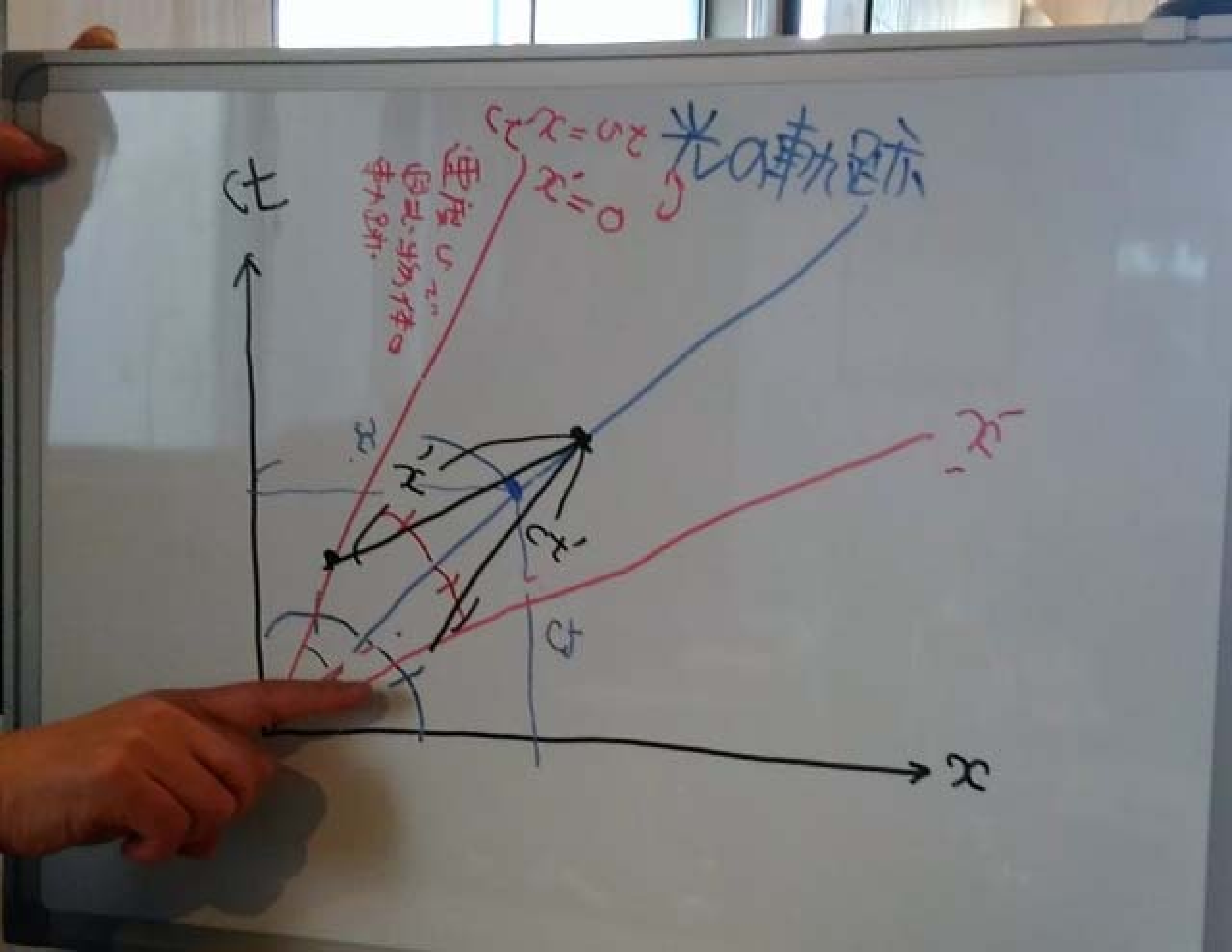

光の軌跡
ct
x
O
ct


光の軌跡
ct
θ
ct
O
x



光の軌跡
ct'
x=vt
x'=0
速度vで運動物体の軌跡
ct
x
O


光の軌跡
ct'
x = vt
x' = 0
速度vで動く物体の軌跡
ct
x'
ct
x
O

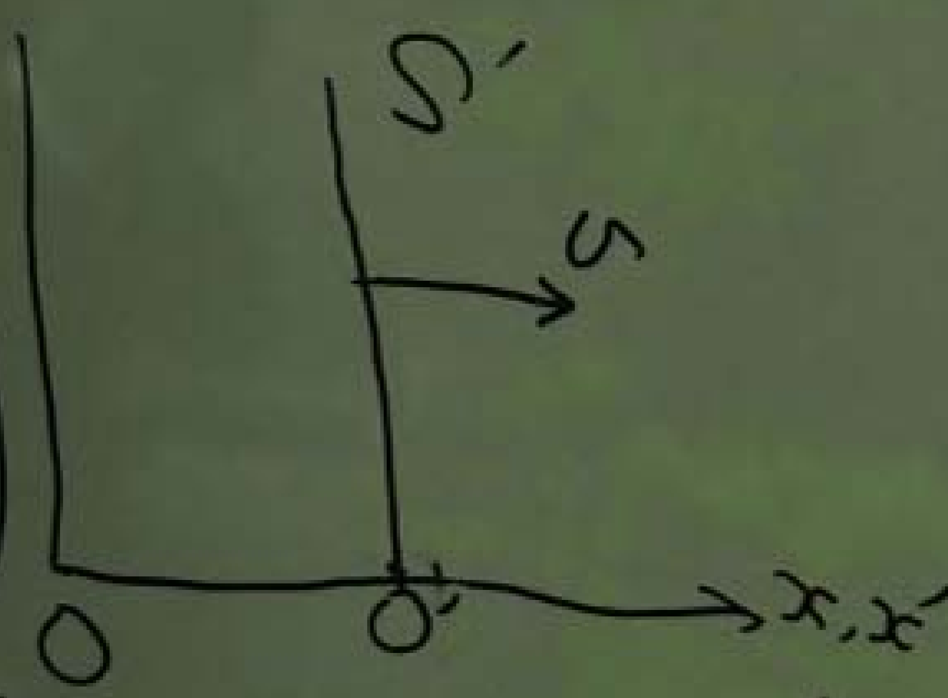


ct
x = vt
x' = 0
光の軌跡
x'
ct'
x

S（静止系）

S'（Sに対して速度vで走っている系）

$$\boxed{x'=0 \Leftrightarrow x=vt}$$

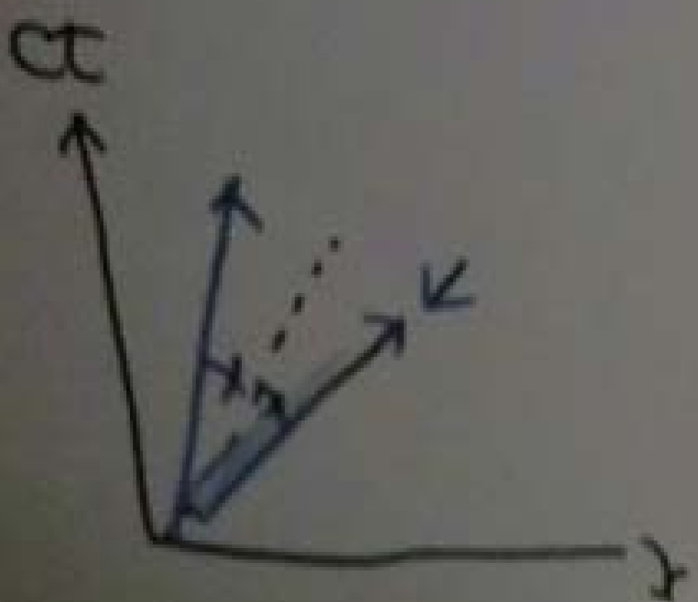


$$x' = A\cdot(x-vt) \longleftarrow \text{に定式}$$

$$x' = A\left(x - \frac{v}{c}ct\right) \quad \text{— ①}$$

$$ct' = A\left(ct - \frac{v}{c}x\right) \quad \text{— ②}$$

$ct \leftrightarrow x$
とりかえる
(対称性)



$$x' = A\left(x - \frac{v}{c}ct\right) \quad —①$$

$$ct' = A\left(ct - \frac{v}{c}x\right) \quad —②$$

S'がSに対してvで運動している

$\Leftrightarrow$ SがS'：$-v$

$$x = A\left(x' + \frac{v}{c}ct'\right) \quad —①'$$

$$ct = A\left(ct' + \frac{v}{c}x'\right) \quad —②'$$

$$x' = A\left(x - \frac{v}{c}ct\right) \quad \text{— (1)}$$

$$ct' = A\left(ct - \frac{v}{c}x\right) \quad \text{— (2)}$$

$$A\left[A(x - vt) + \frac{v}{c}A\left(ct - \frac{v}{c}x\right)\right]$$

$$x = A\left(x' + \frac{v}{c}ct'\right) \quad \text{— (1)}$$

$$\boxed{\begin{aligned} x' &= A\left(x - \frac{v}{c}ct\right) \quad \text{— (1)} \\ ct' &= A\left(ct - \frac{v}{c}x\right) \quad \text{— (2)} \end{aligned}}$$

$$x = A^2\left[\left(x - \cancel{vt}\right) + \frac{v}{c}\left(\cancel{ct} - \frac{v}{c}x\right)\right]$$

$$= \boxed{A^2\left(1 - \frac{v^2}{c^2}\right)}\, x$$

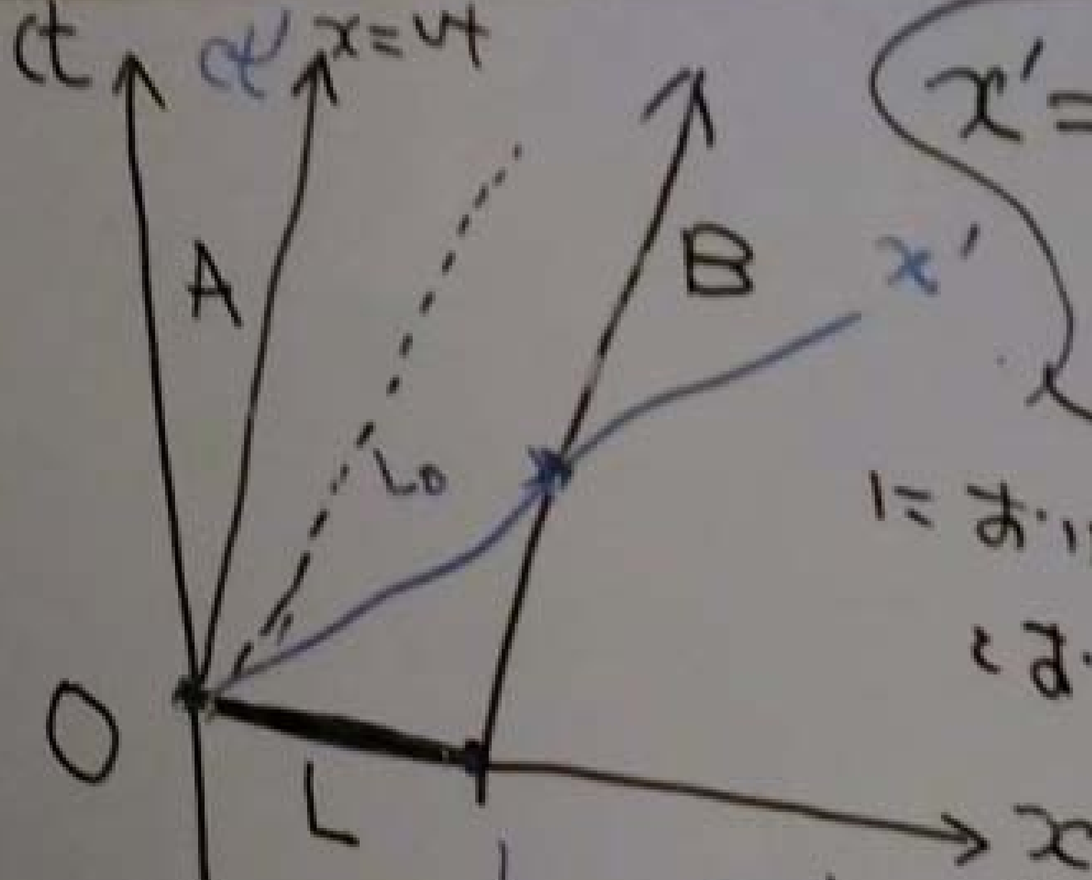
$$A^2\left(1 - \frac{v^2}{c^2}\right) = 1$$

$$\therefore\ A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{— ①} \\ ct' &= \frac{ct - \frac{v}{c}x}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{— ②} \end{aligned}}$$

Lorentz 变换

$$\left(\begin{aligned} y' &= y \\ z' &= z \end{aligned} \right)$$



$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

において $t=0$

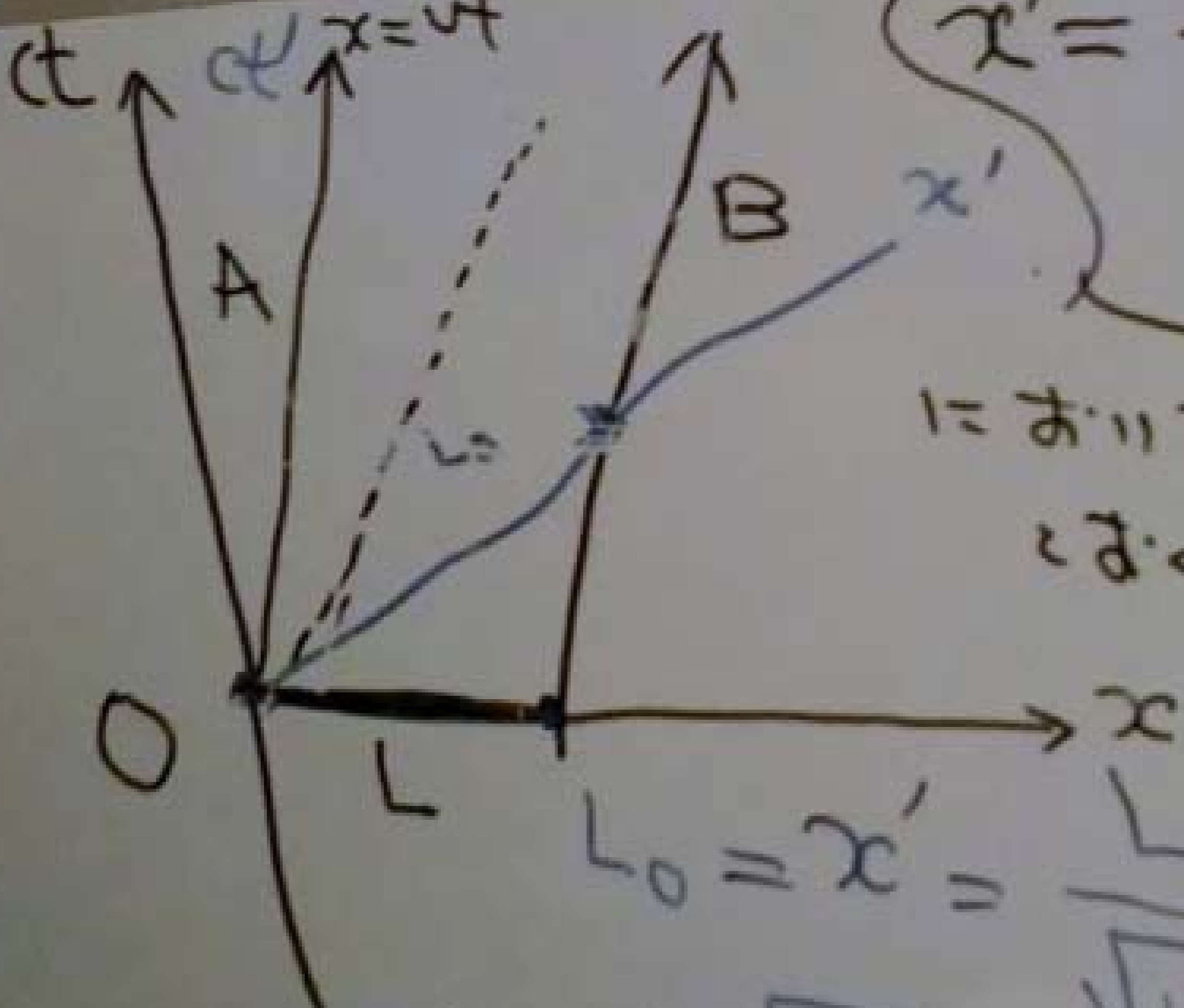
とおくと

$$L_0 = x' = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} > L$$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} < L_0$$

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

において $t=0$ とおくと



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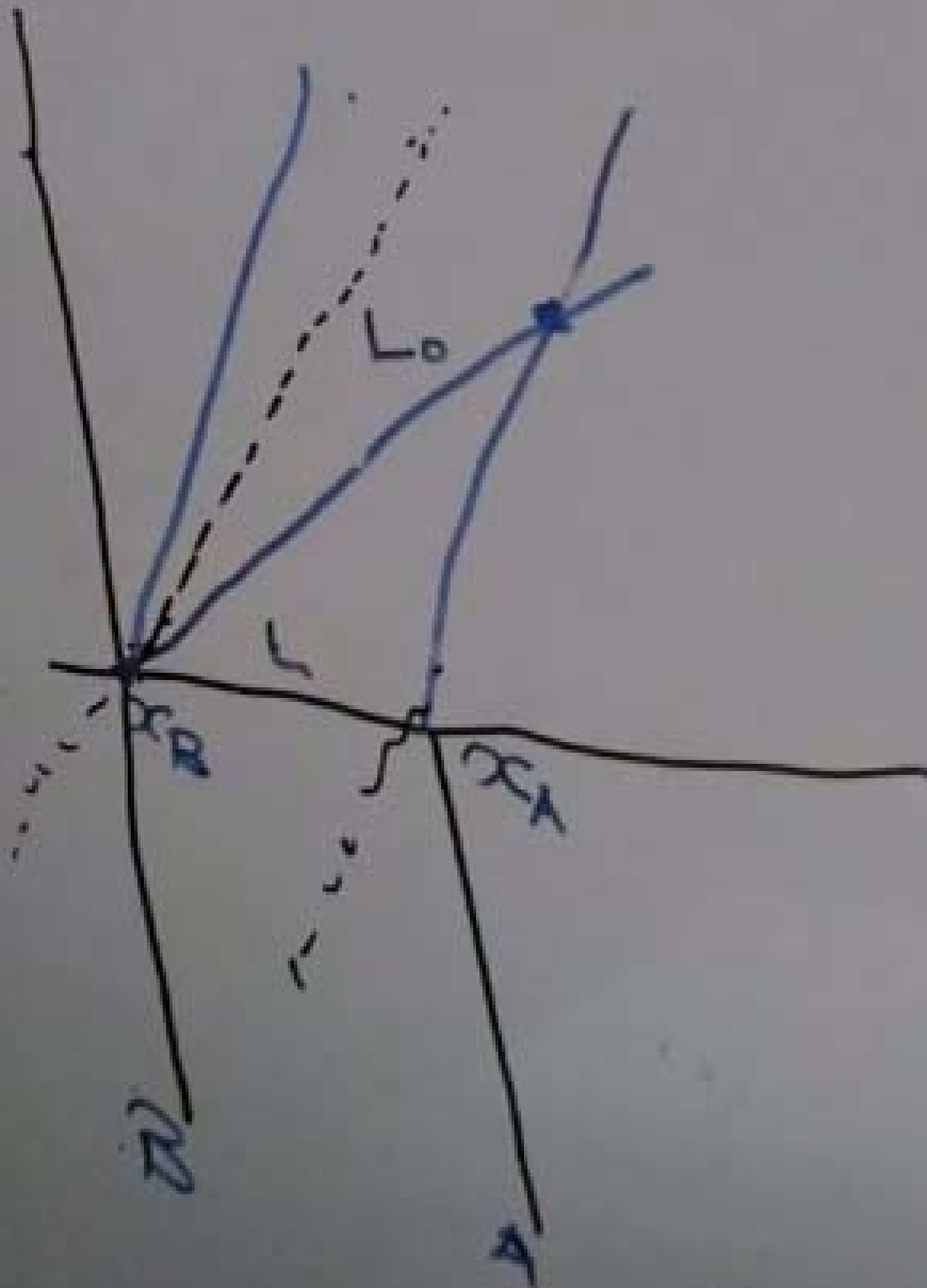

L_0
L
x_B
x_A
B
A